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6.3 The use of the Gamma Model to estimate magazine issue accumulation

INTRODUCTION

In 1977 *Newsweek* asked Audits & Surveys to develop a method to determine what percentage of its average issue audience grows from its date of publication. A major nationwide retailer was thinking of using news weeklies to advertise special promotions and wanted to know how many people would have read the magazines while the promotion was still in effect. Our task was to find and test a procedure that would be both reliable and economical.

THE PROBLEM

When an issue of a weekly, bi-weekly or monthly magazine is published it is known that the issue will generate an audience. A great deal is known about the number of readers per copy and how this value varies depending upon the nature of the magazine. A great deal is known also about how the audiences of different magazines accumulate across issues.

Very little is known however, about the rate of accumulation of readers within a specific issue. In conducting audience studies by means of through-the-book technique, it is assumed that the optimum age of issue for measuring the number of readers is four-and-one-half weeks for a weekly publication and about ten weeks for a monthly. These points have been used based upon assumptions of the cumulation curve and the curve of forgetting time periods.

There are no data that indicate what percentage of these readers has been reached on the first day after publication, the second day and on each succeeding day. While the overall growth pattern should be fairly stable, the daily growth curve may be erratic due to the occurrence of special days set aside for reading.

THE DIRECT OR BRUTE FORCE APPROACH

Estimates of daily growth may be obtained by selecting a representative sample and asking respondents about their reading occasions of a particular issue of a news weekly 'yesterday'. If one of the yesterday occasions was the first time a respondent had looked into the test issue, then the respondent would be counted as a new reader as of that day.

However, if this method is used an extremely large sample would be required. To illustrate, the following hypothetical data have been postulated. For a given news weekly where the percentage of population reading an average issue is 12.7, the growth of its audience might be as indicated in the following two tables. **Table 1** shows the percentage of the population that becomes readers of an average issue from the first day after publication to the 35th day.

If these percentages are accumulated, then the daily growth curve is obtained. This is shown in **Table 2**.

Using this example, it can be seen from **Table 2** that 54% of a news weekly's audience (6.9 of 12.7) is reached in seven days, 73% at the end of the second week, 85% at the end of the third week, etc.

TABLE 1
Percentage of population becoming readers of a news weekly (hypothetical data) by day

Day	*Week 1*	*Week 2*	*Week 3*	*Week 4*	*Week 5*
First	2.00	.45	.25	.17	.13
Second	1.33	.40	.24	.17	.13
Third	1.00	.37	.22	.16	.12
Fourth	.80	.33	.21	.15	.12
Fifth	.67	.31	.20	.15	.12
Sixth	.57	.29	.19	.14	.12
Seventh	.50	.27	.18	.14	.12
Total	6.87	2.42	1.49	1.08	.86

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TABLE 2
Audience accumulation within an average issue of a news weekly percentage of population (hypothetical data) by day

Day	*Week 1*	*Week 2*	*Week 3*	*Week 4*	*Week 5*
First	2.0	7.3	9.5	11.0	12.0
Second	3.3	7.7	9.8	11.1	12.1
Third	4.3	8.1	10.0	11.3	12.2
Fourth	5.1	8.4	10.2	11.4	12.4
Fifth	5.8	8.7	10.4	11.6	12.5
Sixth	6.4	9.0	10.6	11.7	12.6
Seventh	6.9	9.3	10.8	11.9	12.7

TABLE 3

Extent of accumulation	*Audience estimate (percentage)*	*Sample size*	*Standard error of estimate (percentage points)*	*Relative error*
1 day	2.0	2,000	.31	16%
2 days	3.3	4,000	.40	12%
3 days	4.3	6,000	.46	11%
4 days	5.1	8,000	.50	10%
5 days	5.8	10,000	.53	9%
6 days	6.4	12,000	.56	9%
7 days	6.9	14,000	.58	8%
14 days	9.3	28,000	.68	7%
35 days	12.7	70,000	.79	6%

While the concept is straightforward, the operational difficulty with the 'single day' approach is the need for very large samples.

Suppose that, for each day over a period of 35 days, a sample of 2000 persons was selected, and a determination is made of the percentage of the population becoming readers. Using unrestricted random sampling, the sampling errors of some of the estimates in **Table 2** would be as shown in **Table 3**.

If the audience accumulation were estimated through 14 days by the 'single day' method, it would take 2000 interviews per day. Thus, 28,000 interviews would be required and the estimates would be subject to relatively large sampling errors. If accumulation was obtained by this method for 35 days then 70,000 interviews would be required.

The size of sample may be reduced by instead of studying a single issue of a news weekly for 35 days, five issues could be used in a single interview. In this approach the interviewer would carry five issues and if reading was done for the first time yesterday, the respondent would identify the particular issue. In the following week of interviewing, the now current issue would provide estimates for week 2, etc. The interviewing should be done daily and cover a complete publication period – seven days for a weekly and 30 days for a monthly.

THE USE OF A MATHEMATICAL MODEL

Because it was impractical to use the direct approach described above to estimate audience accumulation it was decided to investigate the feasibility of using a mathematical model to describe the phenomenon. If a suitable model could be found, its parameters could be evaluated by means of a sample survey and the accumulation determined.

Recognising the fact there is variability, more or less independent of the particular issue of a magazine, in the day-to-day pattern of reading, in the variability due to

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sampling, etc, the model would be required to describe the general pattern of audience growth. Once the general pattern is derived then, if people tend to read the first time on different weekdays a day of the week adjustment can be made in the same manner used in economic time series when seasonal adjustments are merged with a trend line.

In seeking an appropriate mathematical model we soon discovered that we were dealing with a phenomenon encountered in operations research that deals with survival time and is expressed by a survivorship distribution. Time is the independent variable and the number of units, say electric light bulbs, transistors, reusable milk bottles failing during a given time period is the dependent variable. The same phenomenon is encountered in medical research. As one biostatistician describes it 'Survival times are data that measure the time to response, failure, death, relapse, or the development of a given disease. These times are subject to random variations, and like any random variables, form a distribution'. (Lee, 1980).

The amount of time that elapses between the onset of publication of an issue of a magazine and when a reader first reads it can be considered to be a survival time. The aggregate of all times for all readers of the issue can be described by a survivorship distribution.

All distributions are characterised by a density function, f(x), which describes the relative frequency or probability of occurrence at point x. To describe growth of audience as x varies from nought to infinity, the cumulative distribution F(t) is used. In biomedical research and in operations research the Survival Function which describes the proportion of the population that remains viable at time t is used. This function $S(t) = 1 - F(t)$.

There are a number of different survival density functions that could have been used to describe issue audience accumulation. After examining various possibilities it was decided to use the Gamma function. This distribution has certain properties that are realistic in characterising how readers come into an audience of a magazine as the issue ages. This seemed to be especially true for a weekly news magazine.

The Gamma distribution which is a more general case of the Exponential and Chi square distributions has been widely used in industrial reliability studies and while in the exploratory stage in biomedical research is expected to be an important model. While the distribution has great flexibility, the model itself depends upon only two parameters which can be easily obtained from the mean and variance of the empirical distribution.

In contrast with the extremely large numbers of observations needed to determine issue audience growth by the direct method, relatively few observations are needed to estimate the parameters of the distribution. This makes it possible to obtain cumulative growth patterns for different segments of the reader population.

A convenient form of the Gamma density function suitable for issue audience studies is given below. The variable x is time measured from the time the issue comes out.

$$f(x) = \frac{x^{\alpha-1} \; E \times P\,(-x/\beta)}{\beta^{\alpha}\,\Gamma\,(\alpha)}$$

The cumulative distribution function which is used to measure audience growth is:

$$F(x) = \frac{\int_0^x x^{\alpha-1}\; E \times \rho\,(-x/\beta)\; dx}{\beta^{\alpha}\,\Gamma\,(\alpha)}$$

The distribution is fitted by the Method of Moments since:

$$\text{MEAN} = \alpha\beta \qquad \text{VARIANCE} = \alpha\beta^2$$

Tables of the complete Gamma function are readily available and the following expression given by Deming may be used to evaluate the Incomplete Gamma function.

$$\Gamma_x(n) = \int_0^x x^{n-1}\, e^{-x}\, dx$$

$$= x^n \left\{ \frac{1}{n} - \frac{x}{(n+1)\,1!} + \frac{x^2}{(n+2)\,2!} - \frac{x^3}{(n+3)\,3!} + \ldots \right\}$$

If a computer or a programmable calculator is available then the various integrals may be solved through the use of the Gaussian-Legendre quadrature formula or Simpson's Rule.

THE FIELD TEST

In order to test the applicability of the Gamma function to measure issue audience accumulation a field test was conducted in the Milwaukee ADI from 17 March to 22 April 1977 for a total of 35 days interviewing. A total of 1463 interviews were made.

Because of the limited size of sample it was decided to combine three news weekly magazines – *Newsweek*, *Time* and *US News and World Report* and treat them as a single entity, that is as if the three combined were a single magazine. All three have the same day of issue and deal with similar topics. Interviewing was conducted in person. The interviewer carried the five most recent copies of each of the three magazines. A probability selected sample of 1463 respondents 18 years of age and older, one

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respondent per household, was interviewed. Copies of the magazines were shown to the respondent and it was established whether or not the respondent had read any of the copies yesterday and whether or not that copy was read for the first time yesterday. In addition to establishing first time reading the source of the copy read and demographic characteristics of the respondent were obtained.

Figure 1 shows the accumulation of readers of news weeklies, from day of publication to when the average issue is 19 days old. The broken line depicts the curve of the actual data obtained from the percentages of readers who had first looked into the respective issues 'yesterday'.

On the third and fourth days after publication the actual figures exceed the fitted or smooth figures. This is due to the fact that while these magazines first appear on Monday, most of the news weeklies' readers tend to do their reading in the middle of the week, particularly on Wednesday and Thursday, since the majority of copies are delivered on Tuesday and Wednesday.

In this test the growths of the primary and secondary audiences were obtained and the derived curves appear to be consistent with expectations on how different types of audiences develop (see **Figure 2**).

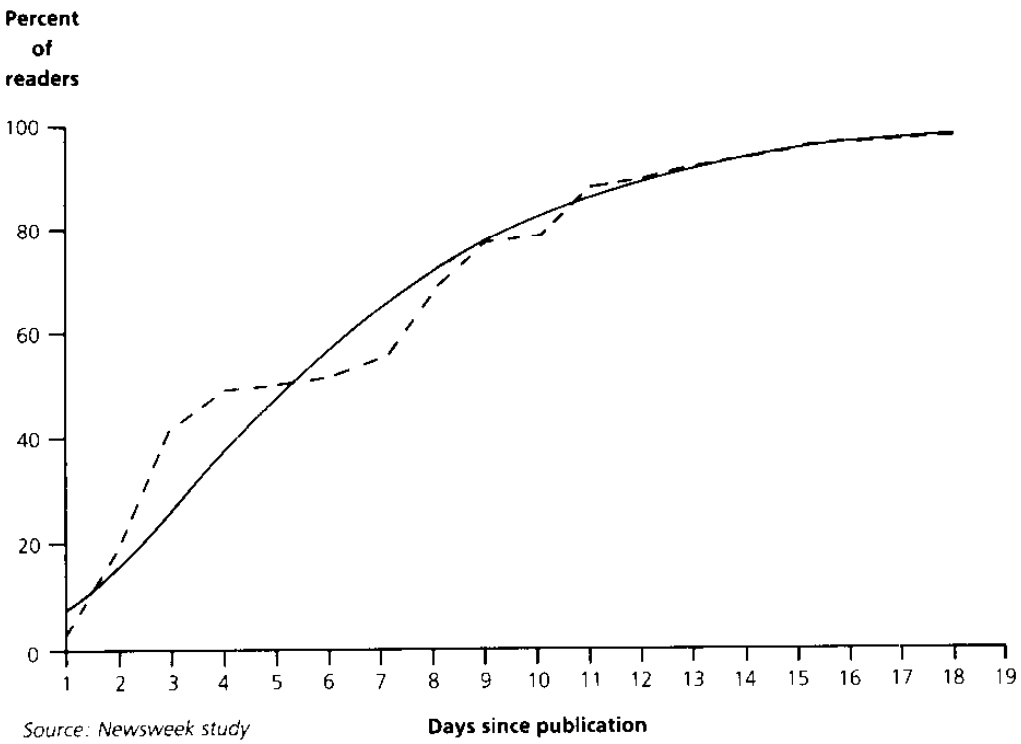
OTHER APPLICATIONS OF THE GAMMA FUNCTION IN THE BEHAVIOURAL SCIENCES

Subsequent to our use of the Gamma function to measure reader accumulation we discovered two other interesting applications.

In a series of 25 mail questionnaire studies, conducted by Bell (Telephone), Canada over a period of time, it was discovered that the number of responses per day since the receipt of the questionnaire followed the Gamma distribution (G Vidgerhous, 1977): this information is being put to use by projecting the distribution after the modal day has passed and knowing the optimum time to send out follow-up mailings (see **Figure 3**).

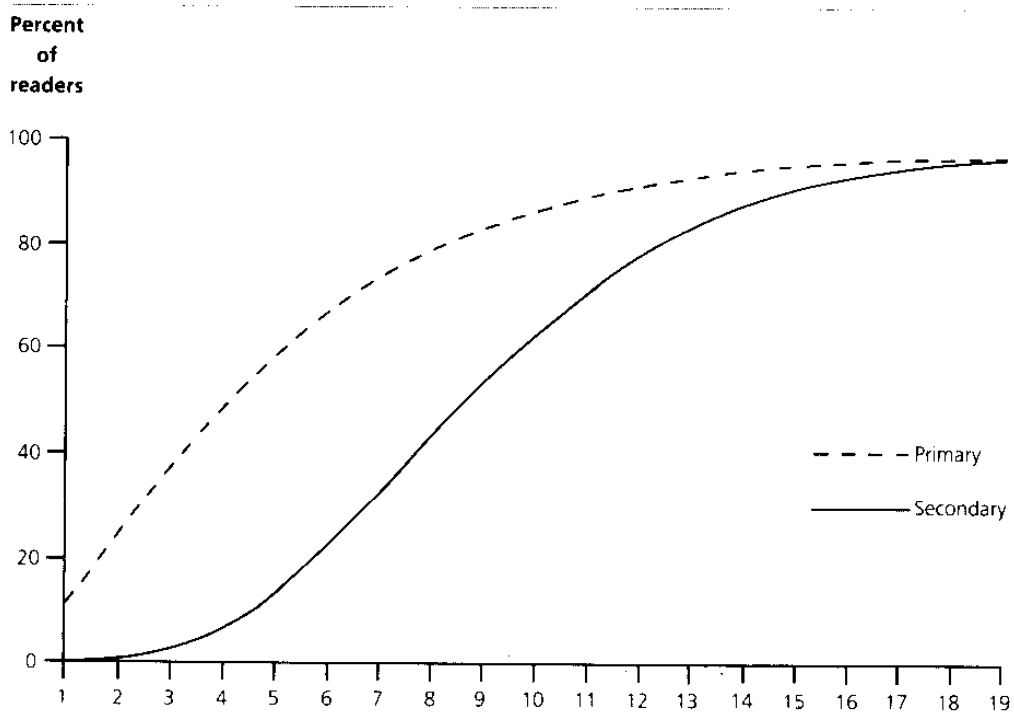
The World Fertility Survey obtains demographic data in many developing countries providing information to test certain theories of nuptiality, the effectiveness of birth

FIGURE 1
Daily audience accumulation, all news weeklies



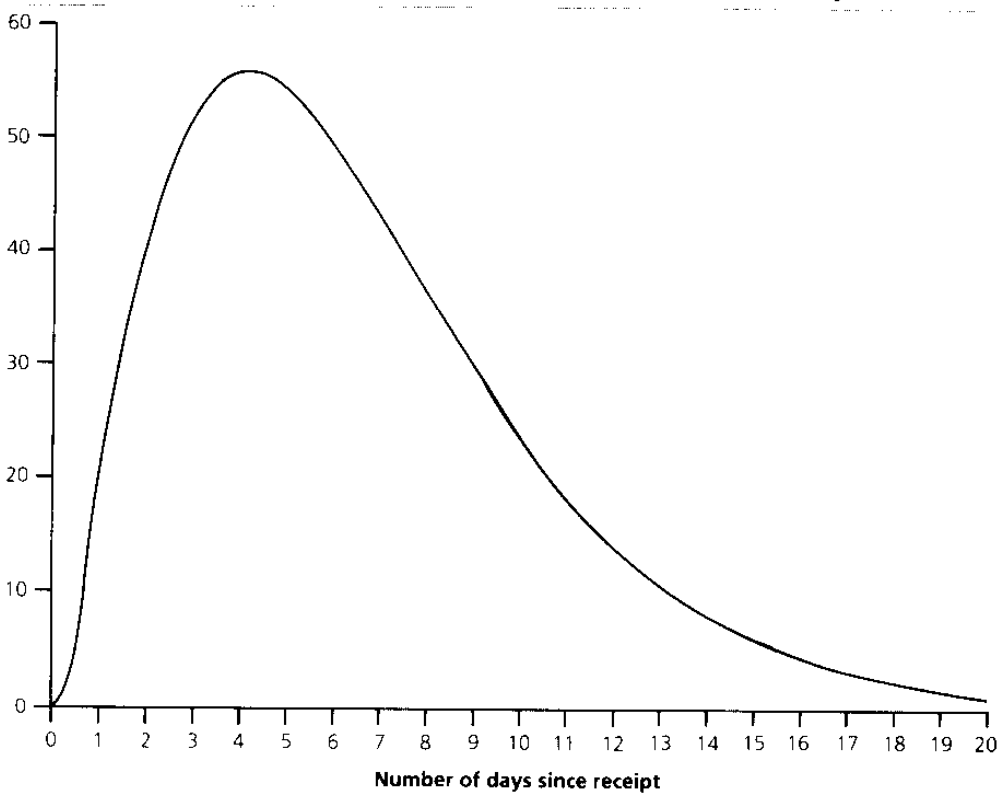
Source: Newsweek study

FIGURE 2
All news weeklies by primary vs. secondary readers



Source: Newsweek study

FIGURE 3
Daily probabilities of questionnaire return (based upon 25 surveys)



Source: G. Vidgerhous, Bell, Canada 1977

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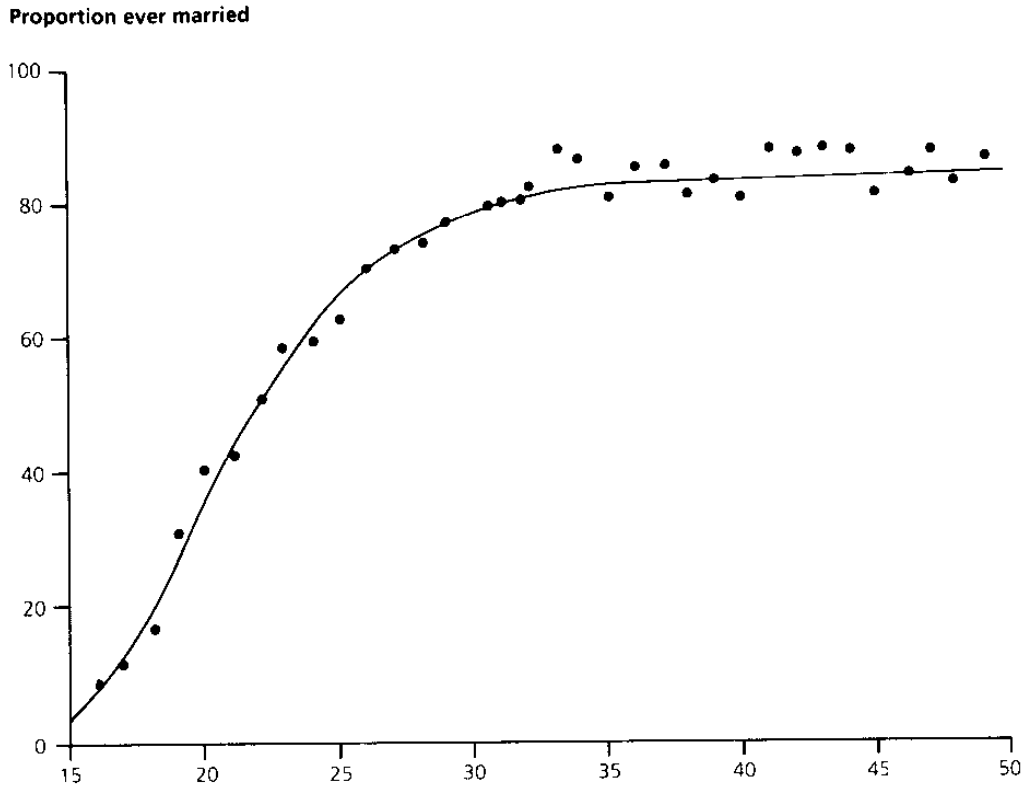
FIGURE 4
Observed and fitted proportions ever married by age (Columbia Household Survey (1976) – sample 12905)



Source: Rodriguez and Trussell

control measures, etc. **Figure 4** is based upon a study of 12,905 women which compared the observed data of age of marriage with fitted values obtained from the cumulative Gamma distribution (Rodriquez and Trussell, 1980).

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Application

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